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# Heat transfer from a square prism to an air stream

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**Abstract**—Experimental investigations on the local and average heat transfer from a square prism to an air stream were carried out in the range of subcritical Reynolds numbers. The average heat transfer coefficients at the angles of attack $\alpha = 0°$ and $45°$ were as high as 40% of the well-known values of Hilpert. The average heat transfer has a minimum value at $\alpha = 12°–13°$ and a maximum one at $\alpha = 20°–25°$. Characteristics of the average and local heat transfer were made clear in connection with the flow characteristics.

## 1. INTRODUCTION

MANY EXPERIMENTAL investigations [1–4] and numerical analyses [5] on the time-averaged and fluctuating fluid forces on a square prism have accumulated in recent years. It is well known that the flow around a square prism can be classified into the following two patterns: at the angle of attack $\alpha$ less than 13°, the shear layer separated from the leading edge does not reattach to the side face; beyond 14° the shear layer reattaches to the side face. Such a square prism is one of the most interesting bluff bodies in connection with the long-standing question of the mechanism of heat transfer in separated region. Nevertheless, there are only two papers so far reported on heat transfer; both are concerned with the average heat transfer of a square prism at the angles of attack $\alpha = 0°$ and 45° [6, 7]. Their results are so familiar as to be quoted in the textbook by Jakob [8]. On the other hand, little is reported about the local heat transfer. To clarify the relation between the fluid flow and heat transfer around a square prism, the author investigated the characteristics of the flow around a prism with angle of attack [9]. In the present study the characteristics of the local heat transfer and of the average one are examined in detail and are correlated to the flow characteristics.

## 2. EXPERIMENTAL APPARATUS AND PROCEDURE

The configuration of a model is illustrated in Fig. 1. Experiments were carried out in a low speed wind tunnel with a working section 400 mm high, 150 mm wide and 800 mm long. Three prisms with side dimensions of 15, 20 and 30 (29.2) mm were adopted as the test models, which were 150 mm long and spanned the tunnel. The free stream velocity ranged from 6 to 30 m/s, and the turbulence level in the working section was less than 0.5%. The Reynolds numbers were in the range $5.6 \times 10^3 \leqslant Re \leqslant 5.6 \times 10^4$. The prisms used for measurement of average heat transfer were made of aluminium, of dimensions 15 and 20 mm. A brass pipe of 12 mm in diameter, containing a heater was inserted in each prism. The surface temperatures around the prism and the temperature of the main flow were measured with the copper–constantan thermocouples of 0.1 mm in diameter. The dimensionless variable, $(\theta_{max} - \theta_{min})/(\theta_{mean} - \theta_\infty)$, was less than 0.01, where $(\theta_{max} - \theta_{min})$ is the difference between the maximum and the minimum values of the surface temperatures and $(\theta_{mean} - \theta_\infty)$ is the difference between the mean surface temperature and the main flow temperature. Consequently, this seems to approximately satisfy the condition of isothermal wall. The average heat transfer from the prism at a constant wall temperature was obtained by using the mean value of surface temperatures. In addition, the whole surface of the prism with the sides of 30 mm fabricated from acrylic resin plates was covered with a stainless steel sheet of 0.02 mm in thickness for the local heat transfer measurement. By applying an alternating current, the local heat transfer coefficient around the prism was measured under the condition of a constant heat flux. The average heat transfer coefficient was evaluated from the distributions of the local heat transfer coefficient. The average heat transfers from the three prisms at the angles of attack from 0° to 45° were measured in the range of Reynolds numbers mentioned above. Furthermore, the results were discussed in connection with the characteristics of the flow around the prism in ref. [9].

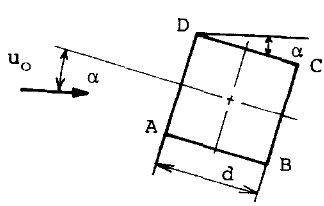



FIG. 1. Flow geometry.

## 3. LOCAL HEAT TRANSFER

### 3.1. *Accuracy of experiment*

It seems profitable to examine the accuracy of the measurement of the local heat transfer from a square prism heated at a constant heat flux. It is well known

## NOMENCLATURE

| | |
|---|---|
| $C_1, C_2$ | constant |
| $C_D, C_L$ | drag and lift coefficients based on $d$ |
| $C_p, C_{pb}$ | pressure coefficient, base pressure coefficient |
| $C'_p$ | fluctuating pressure coefficient $= \Delta p/0.5\rho u_0^2$ |
| $D$ | projected length of a prism in main flow direction |
| $d$ | length of a side of a square prism |
| $d_c$ | diameter of a circular tube of equal exposed surface |
| $f$ | vortex shedding frequency |
| $h, h_m$ | local and average heat transfer coefficient |
| $K$ | base pressure parameter $= (1-C_{pb})^{1/2}$ |
| $L$ | longitudinal length of vortex formation region |
| $m, n$ | constant |
| $Nu, Nu_x$ | local Nusselt number $= hd/\lambda$, $hx/\lambda$ |
| $Nu_m$ | average Nusselt number $= h_m d/\lambda$ |
| $\Delta p$ | root mean square value of fluctuating pressure |
| $Pr$ | Prandtl number |
| $Re, Re_x$ | Reynolds number $= u_0 d/\nu$, $u_\infty x/\nu$ |
| $S$ | Strouhal number $= fd/u_0$ |
| $u_0$ | free stream velocity |
| $u_\infty$ | velocity at the outer boundary layer |
| $x$ | length measured from forward stagnation point. |

Greek symbols

| | |
|---|---|
| $\alpha$ | angle of attack |
| $\lambda$ | thermal conductivity of fluid |
| $\nu$ | kinematic viscosity of fluid |
| $\rho$ | density of fluid. |

that the potential velocity distribution on a wedge with an included angle $\phi$ is given by the following equation:

$$u_\infty = Cx^m, \quad m = \phi/(2\pi - \phi). \tag{1}$$

In the present experiment, the cases of $\alpha = 45°$ and $0°$ correspond to those on wedges with included angles of $\phi = 90°$ and $180°$, respectively. The latter is the case of two-dimensional stagnation flow. The velocity distributions obtained on the basis of the pressure distributions as described later are given by

$$\alpha = 45°: \quad u_\infty/u_0 = 1.02(x/d)^{1/3}, \tag{2}$$

and

$$\alpha = 0°: \quad u_\infty/u_0 = 0.82x/(d/2). \tag{3}$$

These velocity distributions are shown in Fig. 2. The flat plate placed normally to the flow direction, although heated electrically at constant heat flux, manifested a uniform surface temperature. The local heat transfer coefficient on the front surface of a flat plate is given by Katto [10] as follows:

$$\alpha = 0°: \quad Nu_x/\sqrt{Re_x} = 0.57Pr^{0.4}, \tag{4}$$

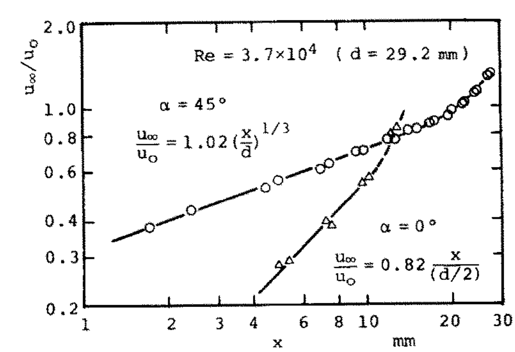



FIG. 2. Velocity distribution of the potential flow in the neighborhood of the stagnation point.

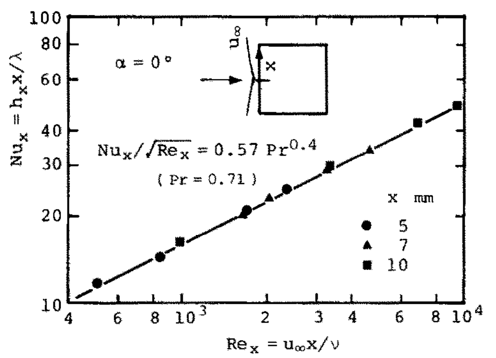



FIG. 3. Local coefficient of heat transfer in the neighborhood of the stagnation point at $\alpha = 0°$.

where the Reynolds number $Re_x$ is defined as $u_\infty x/\nu$. As shown in Fig. 3, the local heat transfer coefficient agrees well with the theoretical value. These measurements are very accurate.

### 3.2. *Local heat transfer*

For four flow patterns described in the previous paper [9], the variations of the distributions of the local heat transfer coefficient around a square prism with $\alpha$ are shown in Figs. 4 (a)–(d). At the angle of $\alpha = 0°$ the coefficients are uniform on both the front and the rear faces, and the value on the rear face is higher than that on the front one. On the side face the value near the trailing edge is higher than that on the rear face and it decreases rapidly toward the region of the leading edge. This suggests that reverse flow exists along the side faces BA and CD. At $\alpha = 10°$ the heat transfer coefficients on the faces BC and CD decrease considerably in comparison with those at $\alpha = 0°$. This causes the reduction of the average heat transfer coefficient in this flow pattern. At $\alpha = 20°$ the coefficient on the side face AB increases remarkably owing to the

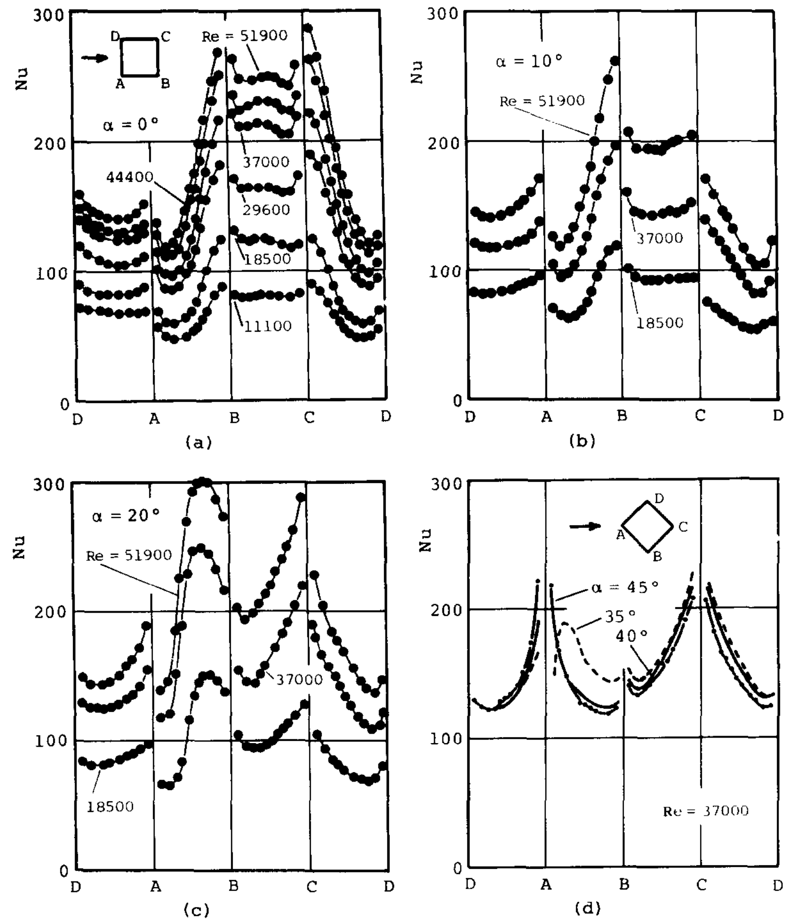



FIG. 4. Local coefficient of heat transfer for a square prism at varying Reynolds numbers.

reattachment of the separated shear layer and the value goes up to maximum at the reattachment point. The values on the faces BC and CD in the separated region are remarkably high, and come to maximum at the corner C. The value increases at the leading edge corner A. In this pattern the local heat transfer coefficient increases over the whole surface. At $\alpha = 35°$ the maximum value of the coefficient is noticed at the point of reattachment on the face AB. Beyond $\alpha = 40°$ the flow becomes a wedge flow, and the local heat transfer coefficients lead to individual maximums at both the front stagnation point A and the rear one C. As the Reynolds number increases, the value of the coefficient on the rear face becomes larger than that on the front face. This is caused by the dependence of the Reynolds number on the heat transfer, that is, $Nu \propto Re^{1/2}$ and $Nu \propto Re^{2/3}$.

## 4. AVERAGE HEAT TRANSFER

### 4.1. *Comparison with other experiments*

The average Nusselt numbers from the prisms at constant wall temperature in the case of angles of attack $\alpha = 0°$ and 45° are shown in Fig. 5. In addition, the results of $d = 30$ mm obtained under the condition of constant heat flux are presented in the figure. These results are compared with those of Reiher [6] and Hilpert [7], which are quoted by Jakob in his textbook [8].

The average heat transfer from 2-dimensional bluff bodies in an air stream is generally presented by

$$Nu_m = C_1 Re^n. \tag{5}$$

The dependence of the Prandtl number on the heat transfer was ascertained by the experiments using different liquids. It is given by the relation

$$Nu_m = C_2 Re^n Pr^{1/3}, \tag{6}$$

where $C_2$ is equal to $1.116C_1$. The constant $C_1$ and the exponent $n$ for a square prism are given in Table 1 together with results by Reiher and Hilpert. The Nusselt and Reynolds numbers are based on the length of a side of a square prism.

Table 1. A comparison of the present result for the constant $C_1$ and the exponent $n$ in equation (5) with those of Reither [6] and Hilpert [7]

| Author | $Re = u_0 d/\nu$ | $\alpha = 0°$ $C_1$ | $n$ | $\alpha = 45°$ $C_1$ | $n$ |
|---|---|---|---|---|---|
| Reiher [6] | 1960– 6000 | 0.149 | 0.699 | 0.238 | 0.624 |
| Hilpert [7] | 3900–78,500 | 0.085 | 0.675 | 0.201 | 0.588 |
| Present exp. | 5600–56,000 | 0.14 | 0.66 | 0.27 | 0.59 |

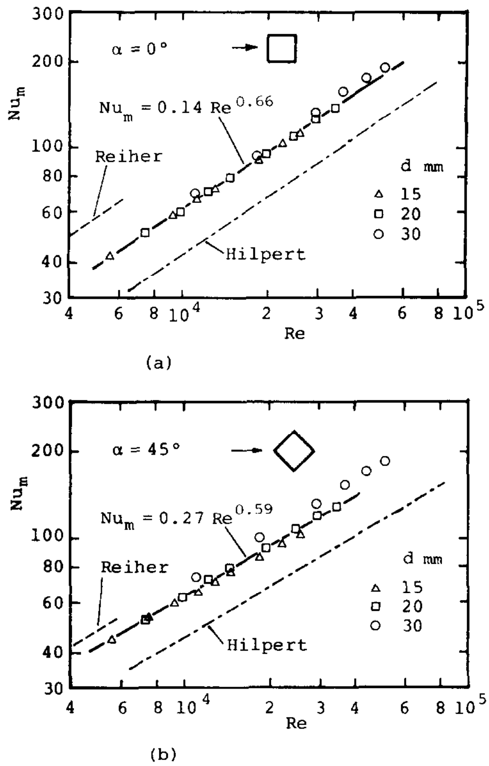



FIG. 5. Comparison between present and other results of average heat transfer from a prism at the angles of attack $\alpha = 0°$ and $45°$ to air stream.

In the paper of Hilpert as well as in the textbook of Jakob [8] and Eckert and Drake [11], the diameter of a circular tube of equal exposed surface $d_c$ is used as the characteristic length. The constant $C_1$ is, therefore, not equal to that in Table 1. At this point, we note an error in several textbooks [12, 13]. The characteristic lengths of the Nusselt number and Reynolds number presented by them were not the above $d_c$, but the projected length in the main flow direction, $D$. These facts are summarized in Table 2.

It is seen from Fig. 5 that at $\alpha = 0°$ there is no effect of the wind tunnel blockage. The present experimental values are intermediate between those of Reiher and Hilpert, and they are as high as 40% of the value of Hilpert. At $\alpha = 45°$ the blockage effect reveals itself, the present experimental values are about 40% higher than those of Hilpert, and are rather near those of Reiher. Then, it is worth noticing that the average heat transfer coefficient on the prism of $d = 30$ mm, measured with a high accuracy at a constant heat flux, is in fair agreement with those on the prisms of $d = 15$ and 20 mm at a constant wall temperature. In the case of $\alpha = 0°$, even under the condition of a constant heat flux, the front and rear faces of the prism resulted in isothermal wall [14, 15]. After all, an accurate measurement of heat transfer on the prisms at a constant wall temperature is made. On the basis of the facts mentioned above, it is natural to say that the values of heat transfer coefficient obtained by Hilpert, which is often referred to in many textbooks, are 40% lower than its true values. But, the exponents $n$ evaluated from the data at $\alpha = 0°$ and $45°$ are identical with those of Hilpert.

In conclusion, in the range of $5.6 \times 10^3 \leqslant Re \leqslant 5.6 \times 10^4$, the average heat transfer coefficient from a square prism at a constant wall temperature to an air stream can be expressed by the following equations

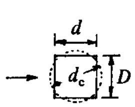
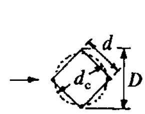
$$\alpha = 0°; \quad Nu_m = 0.14\, Re^{0.66}, \tag{7}$$

$$\alpha = 45°; \quad Nu_m = 0.27\, Re^{0.59}. \tag{8}$$

*4.2. Variation of heat transfer with angle of attack*

Figure 6 shows the variation of the average heat transfer from the prisms to an air stream with angles of attack. Up to the angle of $\alpha = 12°$, the average Nusselt number decreases with increasing $\alpha$. Beyond $\alpha = 13°$ the Nusselt number increases with increasing $\alpha$, and the value at $\alpha = 15°$ is equal to that at $\alpha = 0°$. Then at $\alpha = 20°–25°$ the value reaches to a maximum, and above 25° it decreases gradually with increasing $\alpha$. This

Table 2. The constants in equations (5) and (6) for the heat transfer from a square prism quoted in various textbooks according to Hilpert results [7]

| Geometry | Authors | Quotation | Characteristic length | $Re$ | $C_1$ | $C_2$ | $n$ |
|---|---|---|---|---|---|---|---|
|  | Hilpert [7] | | $d_c$ | 5000–100,000 | 0.092 | | 0.675 |
| | Jakob [8]<br>Eckert & Drake [11] | [7] | $d_c = \frac{4}{\pi} d$ | 5000–100,000 | 0.092 | | |
| | Present author | [7], [8] | $d$ | 3900– 78,500 | 0.085 | 0.094 | 0.675 |
| | Knudsen & Katz [12]<br>Holman [13] | [11]<br>[8] | $D = d$ | 5000–100,000 | 0.092 | 0.102 | |
|  | Hilpert [7] | | $d_c$ | 5000–100,000 | 0.222 | | 0.588 |
| | Jakob [8]<br>Eckert & Drake [11] | [7] | $d_c = \frac{4}{\pi} d$ | 5000–100,000 | 0.222 | | |
| | Present author | [7], [8] | $d$<br>$D = \sqrt{2}d$ | 3927– 78,500<br>5550–111,000 | 0.201<br>0.232 | 0.223<br>0.257 | 0.588 |
| | Knudsen & Katz [12]<br>Holman [13] | [11]<br>[8] | $D = \sqrt{2}d$ | 5000–100,000 | 0.222 | 0.246 | |

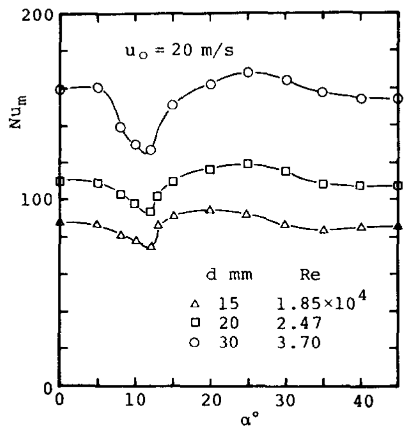



FIG. 6. Variation of average heat transfer with angle of attack.

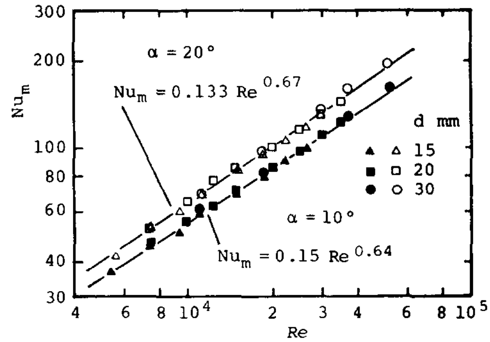



FIG. 7. Correlations of $Nu$ vs $Re$ for $\alpha = 10°$ and 20°.

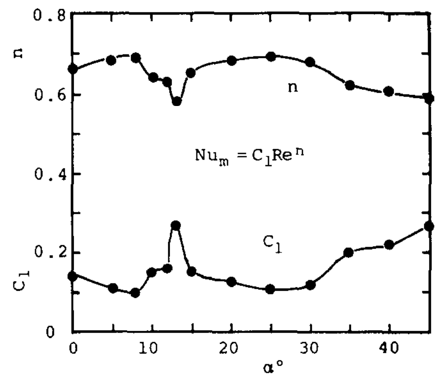



FIG. 8. Variations of constant $C_1$ and exponent $n$ of equation (5).

variation is similar to those of flow characteristics as described later. Consequently, the behavior of the average heat transfer are closely related to that of the shear layer separated at the leading edge of the prism. At angles less than $\alpha = 13°$ the shear layer does not reattach onto the side face of the prism. Beyond $\alpha = 13°$ it reattaches onto the side face, and above $\alpha = 35°$ the flow resembles that about a wedge. The average Nusselt numbers at $\alpha = 10°$ and 20° are shown in Fig. 7 as a function of Reynolds number, and can be presented in the same form as equation (5):

$$\alpha = 10°: \quad Nu_m = 0.15\,Re^{0.64}, \tag{9}$$

$$\alpha = 20°: \quad Nu_m = 0.133\,Re^{0.67}. \tag{10}$$

The variations in the constant $C_1$ and the exponent $n$ in equation (5) for the average heat transfer from a prism with $\alpha$ are shown in Fig. 8. The exponent $n$ has a minimum at $\alpha = 12°$ and a maximum at $\alpha = 25°$. The variation of the exponent $n$ with $\alpha$ is found to coincide in tendency with that of the average heat transfer. On the other hand, the constant $C_1$ decreases with an increase in $n$. It is closely related to $n$ as defined by $C_1 = 0.80 - n$.

Substituting the values of $C_1$ and $n$ illustrated in Fig. 8 into equation (5), the average heat transfer coefficient from a prism at the angle of attack can be estimated.

### 4.3. *Correlations between heat transfer and flow characteristics*

The flow around a prism was visualized using a smoke wind tunnel, and the instantaneous photographs obtained are shown in Fig. 9. The variations of the base pressure parameter ($K$), Strouhal number ($S$)

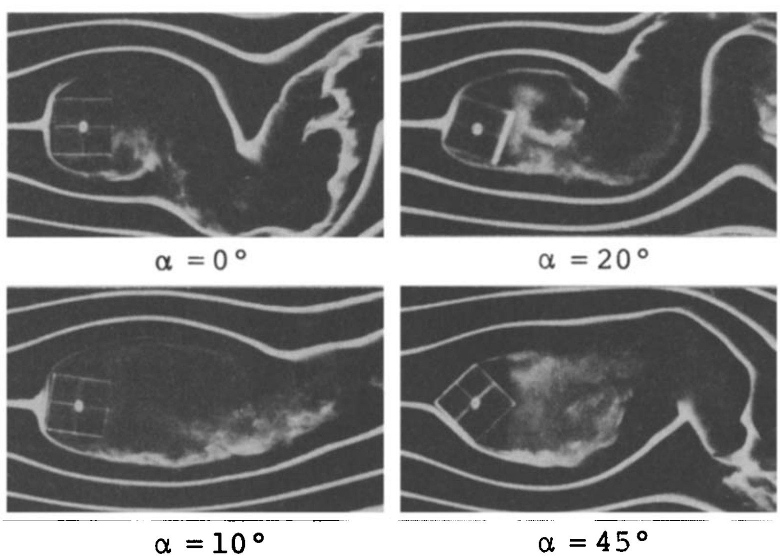



FIG. 9. Visualization of the flow around a prism at $Re$ = 11,000.

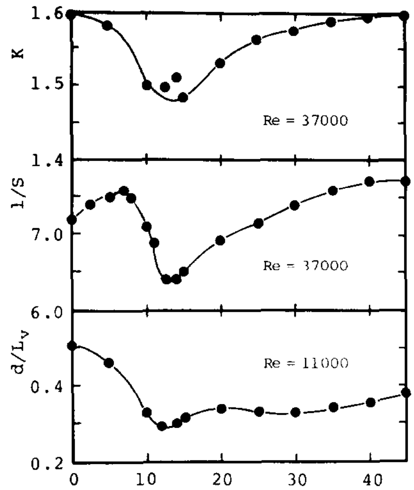



FIG. 10. Variations of base pressure parameter, Strouhal number and vortex formation region with angle of attack $\alpha$.

and longitudinal length of vortex formation region ($L_v$) with attack angle $\alpha$ are displayed in Fig. 10 (where $L_v$ denotes the distance from the center of the prism to the point of intersection of the shear layer separated from both sides of the prism). The distance $L_v$ is evaluated from the above flow patterns. Similarly, Fig. 11 indicates the variation of the fluctuating pressure coefficient ($C_p'$) with $\alpha$, and Fig. 12 the variations of the mean drag coefficient ($C_D$) and lift one ($C_L$) with $\alpha$. From the previous report [9], the flow patterns around a square prism can be classified according to the angles of attack $\alpha$, and are summarized as:

(I) $0° \leqslant \alpha \leqslant 5°$; perfect separation type symmetric flow,
(II) $5° < \alpha \leqslant 13°$; perfect separation type unsymmetric flow,
(III) $14° \leqslant \alpha \leqslant 35°$; reattachment flow type,
(IV) $35° < \alpha \leqslant 45°$; wedge flow type.

These flow characteristics present adequate correlation with each other, and show a similar tendency to that of heat transfer. At the angle of $\alpha = 13°$, the vortex

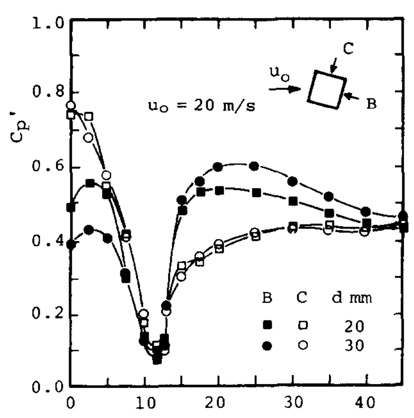



FIG. 11. Fluctuating pressure coefficient $C_p'$ vs angle of attack $\alpha$.

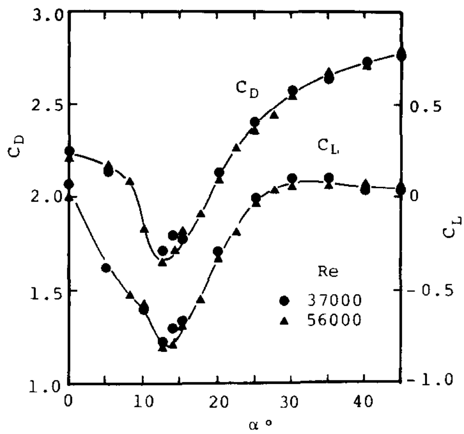



FIG. 12. Drag coefficient $C_D$ and lift coefficient $C_L$ vs angle of attack $\alpha$.

formation region $L_v$ is shifted to the most downstream side and the wake width has a minimum. In this case, every value of $K$, $C_D$, $C_L$ and $C_p'$ has a minimum and that of $S$ has a maximum. The fluctuating pressure coefficient $C_p'$ closely resembles the characteristics of heat transfer in its variations with the angle of attack.

## 5. CONCLUSIONS

Experimental investigations on the local and average heat transfer from a square prism to an air stream were carried out in the range $5.6 \times 10^3 \leqslant Re \leqslant 5.6 \times 10^4$. The main results are summarized as follows:

(1) There is no difference between the average heat transfer from the prism under the condition of a constant wall temperature and that under a constant heat flux.

(2) The average heat transfer coefficients at the angles of attack $\alpha = 0°$ and 45° are as high as 40% of the well known values of Hilpert, and are represented as $\alpha = 0°: Nu_m = 0.14\,Re^{0.66}$; $\alpha = 45°: Nu_m = 0.27\,Re^{0.59}$.

(3) The variation of the average heat transfer coefficient with the angles $\alpha$ has a minimum value at 12°–13° and has a maximum at 20°–25°.

(4) The heat transfer at the angles from 0° to 45° can be estimated by the equation $Nu_m = C_1\,Re^n$, where the constants $C_1$ and $n$ are illustrated in Fig. 5.

(5) The heat transfer is closely connected with the characteristics of the flow around the prism, that is, it increases with individual increases in every value of $d/L_v$, $1/S$, $K$, $C_D$, $C_L$ and $C_p'$.

(6) The distribution of the local heat transfer coefficient is characterized by the flow patterns connected with the reattachments of the separated shear layer and of the reverse flow in a wake depending upon the angle of attack.

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## TRANSFERT THERMIQUE AUTOUR D'UN PRISME CARRE DANS UN COURANT D'AIR

**Résumé**—Des expériences sur le transfert thermique local et global d'un prisme carré dans un courant d'air, pour le domaine des nombres de Reynolds subcritiques. Les coefficients moyens de convection aux angles d'attaque $\alpha = 0$ et 45° sont supérieurs de 40% aux valeurs classiques d'Hilpert. Le coefficient moyen a une valeur minimale à $\alpha = 12$–13° et une valeur maximale à $\alpha = 20$–25°. Des caractéristiques des transferts moyens et locaux sont clarifiés en relation avec les caractéristiques de l'écoulement.

## DER WÄRMEÜBERGANG VON EINEM QUADRATISCHEN PRISMA AN EINEN LUFTSTROM

**Zusammenfassung**—Der örtliche und mittlere Wärmeübergang von einem quadratischen Prisma an einen Luftstrom wurde im Bereich unterkritischer Reynoldszahlen experimentell untersucht. Die mittleren Wärmeübergangskoeffizienten bei den Anströmwinkeln $\alpha = 0°$ und $\alpha = 45°$ betrugen 40% der wohlbekannten Werte von Hilpert. Der mittlere Wärmeübergang hat einen Minimalwert bei $\alpha = 12$–13° und einen Maximalwert bei $\alpha = 20$–25°. Das Verhalten des mittleren und örtlichen Wärmeübergangs wurde anhand der Strömungscharakteristik erklärt.

## ТЕПЛООТДАЧА ОТ ПРЯМОУГОЛЬНОЙ ПРИЗМЫ К ВОЗДУШНОМУ ПОТОКУ

**Аннотация**—Экспериментально исследованы локальный и средний коэффициенты теплообмена прямоугольной призмы с воздушным потоком в диапазоне сверхкритических чисел Рейнольдса. Средние коэффициенты теплообмена при углах атаки $\alpha = 0°$ и 45° достигали 40% от хорошо известных значений Гилберта. Минимальное значение среднего коэффициента теплообмена отмечалось при $\alpha = 12°$–13°, а максимальное – при $\alpha = 20°$–25°. Поведение характеристик среднего и локального коэффициентов теплообмена становится понятным при согласованном их рассмотрении с характеристиками обтекания.